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Analysis of Various Reliability Procedures for a Comparable Redundant Gas Separator

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ABSTRACT:
In this paper, the author has considered the gas separator plant for estimation of its important reliability parameters. In this gas separator plant, there are four subsystems, named here as A, B, C and D, connected in series. The subsystem A is a pressure shutdown valve (PSD) and it is responsible to collect input for separation process. This is an automatic valve and it works by the pressure of input. The second subsystem B is a pressure logic controller (PLC) unit and it controls the working of PSD valve. The third subsystem C is gas separator unit and it separates the useful gas from unused gases. After separation of gases, it exhausted the unused gases and allows moving the useful gas to the next subsystem. The fourth subsystem D is again a PSD valve.

KEY WORDS*:* reliability parameters, pressure shutdown valve

INTRODUCTION:

In this model, the author has taken one parallel redundant gas separator to improve reliability measures of the whole system. On failure of any one of the gas separators, the whole system works in reduced efficiency state. The whole system may fail due to failure of any one of the four subsystems. The block-diagram of considered system has been shown in fig-1. All failures follow exponential time distribution and all repairs follow general time distribution. The system has to wait for repair, in case, the entire subsystem C is failed, otherwise repair facilities are always available. Supplementary variable technique has been used for mathematical formulation of the model. Flow of states has been shown in fig-2. Head-of-line policy has been adopted for repair purpose. This policy is nothing but the first come first served policy. Laplace transform has been used to solve the mathematical model of the considered system. Steady-state behaviour of the system and a particular case, when repairs follow exponential time distribution, have also been computed to improve practical utility of the model. Availability function, reliability function and M.T.T.F. of considered system have been computed. A numerical illustration and its graphical demonstration have been appended in the end to show the importance of the results, obtained in this study.

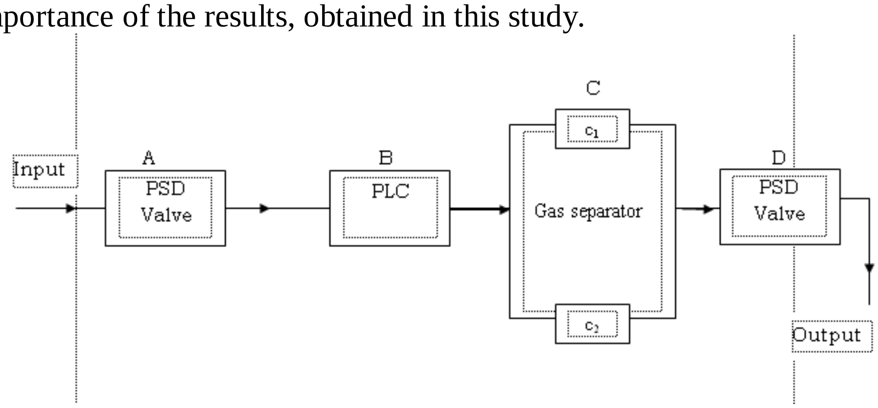

Fig-1 : Block-diagram of gas separator

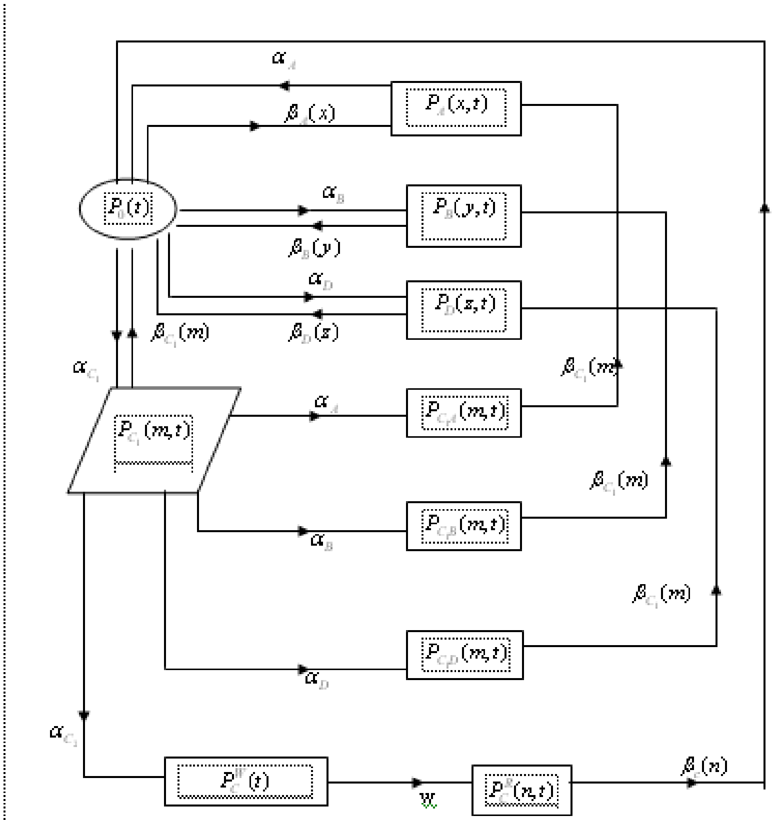

Fig-2 : Flow of states of considered system

NOMANCLATURE:

I have used the following nomenclature in this:

α_i : Failure rate of i^{th} unit/subsystem.

w : Waiting rate for repair.

$\beta_i(j)\Delta$: First order probability that i^{th} repair will be repaired in the time limit $(j, j+\Delta)$, conditioned that it was not repaired up to the time j.

$P_0(t)$: Pr {System is operable}.

$P_i(j,t)\Delta$: Pr {System suffers with i^{th} failure}.Elapsed repair time lies in the interval $(j, j+\Delta)$, where i= A,B,D and j= x,y,z respectively.

$P_{C_1 i}(m,t)\Delta$: Pr {System suffers with i^{th} failure while gas separator C_1 has already failed}. Elapsed repair time for C_1 lies in the interval $(m, m+\Delta)$., i= A,B,D.

$P_C^W(t)$: Pr {System is failed due to failure of subsystem C and is waiting for repair}.

$P_C^R(n,t)$: Pr {System is failed due to failure of subsystem C and is ready for repair}.Elapsed repair time lies in the limit $(n, n+\Delta)$.

$S_i(j)$: $\beta_i(j)\exp.\left\{-\int \beta_i(j)dj\right\}, \forall\ i\ and\ j$

$D_i(j)$: $\left[1-\overline{S}_i(j)\right]/ j, \forall\ i\ and$ j

M_i : $-\overline{S}'_i(0)$ and is mean time to repair i^{th} failure.

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FORMULATION OF MATHEMATICAL MODEL:

By using elementary probability considerations and limiting procedure, we obtain the following set of difference-differential equations, which is continuous in time, discrete in space, governing the behaviour of considered model:

$$\left[\frac{d}{dt}+\alpha_A+\alpha_B+\alpha_D+\alpha_{C_1}\right]P_0(t)=\int_0^\infty P_A(x,t)\beta_A(x)dx+\int_0^\infty P_B(y,t)\beta_B(y)dy+\int_0^\infty P_D(z,t)\beta_D(z)dz+\int_0^\infty P_{C_1}(m,t)\beta_{C_1}(m)dm+\int_0^\infty P_C^R(n,t)\beta_C(n)dn \quad \ldots(1)$$

$$\left[\frac{\partial}{\partial x}+\frac{\partial}{\partial t}+\beta_A(x)\right]P_A(x,t)=0 \quad \ldots(2)$$

$$\left[\frac{\partial}{\partial y}+\frac{\partial}{\partial t}+\beta_B(y)\right]P_B(y,t)=0 \quad \ldots(3)$$

$$\left[\frac{\partial}{\partial z}+\frac{\partial}{\partial t}+\beta_D(z)\right]P_D(z,t)=0 \quad \ldots(4)$$

$$\left[\frac{\partial}{\partial m}+\frac{\partial}{\partial t}+\alpha_A+\alpha_B+\alpha_D+\alpha_{C_2}+\beta_{C_1}(m)\right]P_{C_1}(m,t)=0 \quad \ldots(5)$$

$$\left[\frac{\partial}{\partial m}+\frac{\partial}{\partial t}+\beta_{C_1}(m)\right]P_{C_1A}(m,t)=\alpha_A P_{C_1}(m,t) \quad \ldots(6)$$

$$\left[\frac{\partial}{\partial m}+\frac{\partial}{\partial t}+\beta_{C_1}(m)\right]P_{C_1B}(m,t)=\alpha_B P_{C_1}(m,t) \quad \ldots(7)$$

$$\left[\frac{\partial}{\partial m}+\frac{\partial}{\partial t}+\beta_{C_1}(m)\right]P_{C_1D}(m,t)=\alpha_D P_{C_1}(m,t) \quad \ldots(8)$$

$$\left[\frac{d}{dt}+w\right]P_C^W(t)=\alpha_{C_2}P_{C_1}(t) \quad \ldots(9)$$

$$\left[\frac{\partial}{\partial n}+\frac{\partial}{\partial t}+\beta_C(n)\right]P_C^R(n,t)=0 \quad \ldots(10)$$

BOUNDARY CONDITIONS ARE:

$$P_A(0,t)=\alpha_A P_0(t)+\int_0^\infty P_{C_1A}(m,t)\beta_{C_1}(m)dm \quad \ldots(11)$$

$$P_B(0,t)=\alpha_B P_0(t)+\int_0^\infty P_{C_1B}(m,t)\beta_{C_1}(m)dm \quad \ldots(12)$$

$$P_D(0,t)=\alpha_D P_0(t)+\int_0^\infty P_{C_1D}(m,t)\beta_{C_1}(m)dm \quad \ldots(13)$$

$$P_{C_1}(0,t)=\alpha_{C_1}P_0(t) \quad \ldots(14)$$

$$P_{C_1A}(0,t)=0 \quad \ldots(15)$$

$$P_{C_1B}(0,t)=0 \quad \ldots(16)$$

$$P_{C_1D}(0,t)=0 \quad \ldots(17)$$

$$P_C^R(0,t)=wP_C^W(t) \quad \ldots(18)$$

INITIAL CONDITIONS ARE: …(19)

$P_0(0)=1,$ otherwise all state probabilities are zero at $t=0$.

SOLUTION OF THE MODEL:

Taking Laplace transforms of equations (1) through (18), subjected to initial conditions (19), we obtain:

$$\begin{aligned}(s+\alpha_A+\alpha_B+\alpha_D+\alpha_{C_1})\overline{P}_0(s)=1&+\int_0^\infty \overline{P}_A(x,s)\beta_A(x)dx+\int_0^\infty \overline{P}_B(y,s)\beta_B(y)dy\\&+\int_0^\infty \overline{P}_D(z,s)\beta_D(z)dz+\int_0^\infty \overline{P}_{C_1}(m,s)\beta_{C_1}(m)dm\\&+\int_0^\infty \overline{P}_C^R(n,s)\beta_C(n)dn\end{aligned} \quad \ldots(20)$$

$$\left[\frac{\partial}{\partial x}+s+\beta_A(x)\right]\overline{P}_A(x,s)=0 \quad \ldots(21)$$

$$\left[\frac{\partial}{\partial y}+s+\beta_B(y)\right]\overline{P}_B(y,s)=0 \quad \ldots(22)$$

$$\left[\frac{\partial}{\partial z}+s+\beta_D(z)\right]\overline{P}_D(z,s)=0 \quad \ldots(23)$$

$$\left[\frac{\partial}{\partial m}+s+\alpha_A+\alpha_B+\alpha_D+\alpha_{C_2}+\beta_{C_1}(m)\right]\overline{P}_{C_1}(m,s)=0 \quad \ldots(24)$$

$$\left[\frac{\partial}{\partial m}+s+\beta_{C_1}(m)\right]\overline{P}_{C_1A}(m,s)=\alpha_A\overline{P}_{C_1}(m,s) \quad \ldots(25)$$

$$\left[\frac{\partial}{\partial m}+s+\beta_{C_1}(m)\right]\overline{P}_{C_1B}(m,s)=\alpha_B\overline{P}_{C_1}(m,s) \quad \ldots(26)$$

$$\left[\frac{\partial}{\partial m}+s+\beta_{C_1}(m)\right]\overline{P}_{C_1D}(m,s)=\alpha_D\overline{P}_{C_1}(m,s) \quad \ldots(27)$$

$$(s+w)\overline{P}_C^W(s)=\alpha_{C_2}\overline{P}_{C_1}(s) \quad \ldots(28)$$

$$\left[\frac{\partial}{\partial n}+s+\beta_C(n)\right]\overline{P}_C^R(n,s)=0 \quad \ldots(29)$$

$$\overline{P}_A(0,s)=\alpha_A\overline{P}_0(s)+\int_0^\infty \overline{P}_{C_1A}(m,s)\beta_{C_1}(m)dm \quad \ldots(30)$$

$$\overline{P}_B(0,s)=\alpha_B\overline{P}_0(s)+\int_0^\infty \overline{P}_{C_1B}(m,s)\beta_{C_1}(m)dm \quad \ldots(31)$$

$$\overline{P}_D(0,s)=\alpha_D\overline{P}_0(s)+\int_0^\infty \overline{P}_{C_1D}(m,s)\beta_{C_1}(m)dm \quad \ldots(32)$$

$$\overline{P}_{C_1}(0,s)=\alpha_{C_1}\,\overline{P}_0(s) \qquad \ldots(33)$$

$$\overline{P}_{C_1A}(0,s)=0 \qquad \ldots(34)$$

$$\overline{P}_{C_1B}(0,s)=0 \qquad \ldots(35)$$

$$\overline{P}_{C_1D}(0,s)=0 \qquad \ldots(36)$$

$$\overline{P}_C^R(0,s)=w\overline{P}_C^W(s) \qquad \ldots(37)$$

Now, integrating equation (24) and making use of boundary condition (33), we have

$$\overline{P}_{C_1}(m,s)=\alpha_{C_1}\overline{P}_0(s)\exp.\left\{-\left(s+\alpha_A+\alpha_B+\alpha_D+\alpha_{C_2}\right)m-\int\beta_{C_1}(m)dm\right\}$$

$$\Rightarrow \overline{P}_{C_1}(s)=\alpha_{C_1}\overline{P}_0(s)\frac{1-\overline{S}_{C_1}\left(s+\alpha_A+\alpha_B+\alpha_D+\alpha_{C_2}\right)}{\left(s+\alpha_A+\alpha_B+\alpha_D+\alpha_{C_2}\right)} \qquad \ldots(38)$$

or, $\overline{P}_{C_1}(s)=\alpha_{C_1}\overline{P}_0(s)D_{C_1}(K)$

where, $K=s+\alpha_A+\alpha_B+\alpha_D+\alpha_{C_2}$

Integrating (25) subjected to boundary condition (34),we get

$$\overline{P}_{C_1A}(m,s)e^{sm+\int\beta_{C_1}(m)dm}=T+\int\alpha_A\alpha_{C_1}\overline{P}_0(s)e^{-\left(\alpha_A+\alpha_B+\alpha_D+\alpha_{C_2}\right)dm}$$

where, T is constant of integration.

$$\Rightarrow \overline{P}_{C_1A}(m,s)e^{sm+\int\beta_{C_1}(m)dm}=T+\frac{\alpha_A\alpha_{C_1}\overline{P}_0(s)e^{-\left(\alpha_A+\alpha_B+\alpha_D+\alpha_{C_2}\right)m}}{-\left(\alpha_A+\alpha_B+\alpha_D+\alpha_{C_2}\right)}$$

Putting $m=0$ and $dm=0$ in above expression, we get

$$T=\frac{\alpha_A\alpha_{C_1}\overline{P}_0(s)}{\alpha_A+\alpha_B+\alpha_D+\alpha_{C_2}}$$

$$\therefore \overline{P}_{C_1A}(m,s)=\frac{\alpha_A\alpha_{C_1}\overline{P}_0(s)}{\alpha_A+\alpha_B+\alpha_D+\alpha_{C_2}}\left[1-e^{-\left(\alpha_A+\alpha_B+\alpha_D+\alpha_{C_2}\right)m}\right]e^{-sm-\int\beta_{C_1}(m)dm}$$

Integrating this w.r.t. m from 0 to ∞ we obtain

$$\overline{P}_{C_1A}(s)=\frac{\alpha_A\alpha_{C_1}\overline{P}_0(s)}{(K-s)}\left[D_{C_1}(s)-D_{C_1}(K)\right]$$
$$=\alpha_A A(s)\overline{P}_0(s) \qquad \text{(say)} \qquad ..(39)$$

where, $K=s+\alpha_A+\alpha_B+\alpha_D+\alpha_{C_2}$

Similarly, integrating (26) and (27) with the help of (35) and (36), respectively, we get

$$\overline{P}_{C_1B}(s)=\alpha_B A(s)\overline{P}_0(s) \qquad \ldots(40)$$

and $\overline{P}_{C_1D}(s)=\alpha_D A(s)\overline{P}_0(s)$ …(41)

where, $A(s)=\dfrac{\alpha_{C_1}}{K-s}\left[D_{C_1}(s)-D_{C_1}(K)\right]$

Now equation (28) gives on simplification

$$\overline{P}_C^W(s)=\frac{\alpha_{C_2}\alpha_{C_1}}{(s+w)}D_{C_1}(K)\overline{P}_0(s) \qquad \ldots(42)$$

Integrating (29) subjected to (37) and (42), we get

$$\overline{P}_C^R(n,s)=\frac{w\alpha_{C_1}\alpha_{C_2}}{(s+w)}D_{C_1}(K)\overline{P}_0(s)\exp.\left\{-sn-\int\beta_C(n)dn\right\}$$

$$\Rightarrow \overline{P}_C^R(s)=\frac{w\alpha_{C_1}\alpha_{C_2}}{(s+w)}D_{C_1}(K)D_C(s)\overline{P}_0(s) \qquad \ldots(43)$$

Now, equation (30) becomes by using relevant relations

$$\overline{P}_A(0,s)=\alpha_A\overline{P}_0(s)+\frac{\alpha_A\alpha_{C_1}\overline{P}_0(s)}{(K-s)}\left[\overline{S}_{C_1}(s)-\overline{S}_{C_1}(K)\right]$$

$$=\alpha_A\overline{P}_0(s)[1+B(s)] \qquad \ldots(44)$$

where, $B(s)=\frac{\alpha_{C_1}}{K-s}\left[\overline{S}_{C_1}(s)-\overline{S}_{C_1}(K)\right]$

Integrating equation (21) subjected to (44), we have

$$\overline{P}_A(x,s)=\alpha_A\overline{P}_0(s)[1+B(s)]\exp.\left\{-sx-\int\beta_A(x)dx\right\}$$

$$\Rightarrow \overline{P}_A(s)=\alpha_A\overline{P}_0(s)[1+B(s)]D_A(s) \qquad \ldots(45)$$

Similarly, equation (22) and (23) give on integration

$$\overline{P}_B(s)=\alpha_B\overline{P}_0(s)[1+B(s)]D_B(s) \qquad \ldots(46)$$

and $\overline{P}_D(s)=\alpha_D\overline{P}_0(s)[1+B(s)]D_D(s)$ …(47)

In last, equation (20) gives on simplification by using related expressions:

$$\overline{P}_0(s)=\frac{1}{C(s)}$$

Thus, finally we have the following Laplace transforms of various flow state probabilities:

$$\overline{P}_0(s)=\frac{1}{C(s)} \qquad \ldots(48)$$

$$\overline{P}_A(s)=\frac{\alpha_A D_A(s)}{C(s)}[1+B(s)] \qquad \ldots(49)$$

$$\overline{P}_B(s)=\frac{\alpha_B D_B(s)}{C(s)}[1+B(s)] \qquad \ldots(50)$$

$$\overline{P}_D(s)=\frac{\alpha_D D_D(s)}{C(s)}[1+B(s)] \qquad \ldots(51)$$

$$\overline{P}_{C_1}(s)=\frac{\alpha_{C_1}D_{C_1}(K)}{C(s)} \qquad \ldots(52)$$

$$\overline{P}_{C_1A}(s)=\frac{\alpha_A A(s)}{C(s)} \qquad \ldots(53)$$

$$\overline{P}_{C_1B}(s)=\frac{\alpha_B A(s)}{C(s)} \qquad \ldots(54)$$

$$\overline{P}_{C_1D}(s)=\frac{\alpha_D A(s)}{C(s)} \qquad \ldots(55)$$

$$\overline{P}_C^W(s)=\frac{\alpha_{C_1}\alpha_{C_2}}{(s+w)}\frac{D_{C_1}(K)}{C(s)} \qquad \ldots(56)$$

and $\overline{P}_C^R(s) = \frac{w\alpha_{C_1}\alpha_{C_2}}{(s+w)} \frac{D_{C_1}(K)D_C(s)}{C(s)}$ …(57)

where,

$$K = s + \alpha_A + \alpha_B + \alpha_D + \alpha_{C_2} \quad \text{…(58)}$$

$$A(s) = \frac{\alpha_{C_1}}{K-s}\left[D_{C_1}(s) - D_{C_1}(K)\right] \quad \text{…(59)}$$

$$B(s) = \frac{\alpha_{C_1}}{K-s}\left[\overline{S}_{C_1}(s) - \overline{S}_{C_1}(K)\right] \quad \text{…(60)}$$

and $C(s) = s + \alpha_A + \alpha_B + \alpha_D + \alpha_{C_1} - [1 + B(s)][\alpha_A \overline{S}_A(s) + \alpha_B \overline{S}_B(s) + \alpha_D \overline{S}_D(s)]$

$$-\alpha_{C_1}\overline{S}_{C_1}(K) - \frac{w\alpha_{C_1}\alpha_{C_2}}{(s+w)} D_{C_1}(K)\overline{S}_C(s) \quad \text{…(61)}$$

It is interesting to note that

Sum of equations (48) through (57) = $\frac{1}{s}$ …(62)

STEADY-STATE BEHAVIOUR OF THE SYSTEM:

By using final value theorem in Laplace transform, viz; $\underset{t\to\infty}{Lim}\, P(t) = \underset{s\to 0}{Lim}\, s\overline{P}(s) = P$ (say), provided the limit on L.H.S exists, we obtain the following steady-state behaviour of the system from equations (48) through (57):

$$P_0 = \frac{1}{C'(0)} \quad \text{…(63)}$$

$$P_A = \frac{\alpha_A M_A}{C'(0)}[1 + B(0)] \quad \text{…(64)}$$

$$P_B = \frac{\alpha_B M_B}{C'(0)}[1 + B(0)] \quad \text{…(65)}$$

$$P_D = \frac{\alpha_D M_D}{C'(0)}[1 + B(0)] \quad \text{…(66)}$$

$$P_{C_1} = \frac{\alpha_{C_1} D_{C_1}(K_0)}{C'(0)} \quad \text{…(67)}$$

$$P_{C_1A} = \frac{\alpha_A A(0)}{C'(0)} \quad \text{…(68)}$$

$$P_{C_1B} = \frac{\alpha_B A(0)}{C'(0)} \quad \text{…(69)}$$

$$P_{C_1D} = \frac{\alpha_D A(0)}{C'(0)} \quad \text{…(70)}$$

$$P_C^W = \frac{\alpha_{C_1}\alpha_{C_2} D_{C_1}(K_0)}{w.C'(0)} \quad \text{…(71)}$$

and $P_C^R = \alpha_{C_1}\alpha_{C_2} \frac{D_{C_1}(K_0)M_C}{C'(0)}$ …(72)

where, $M_i = -\overline{S}_i'(0) =$ Mean time to repair i^{th} failure.

$$K_0 = K - s$$

$$A(0) = \frac{\alpha_{C_1}}{K_0}\left[M_{C_1} - D_{C_1}(K_0)\right]$$

$$B(0) = \frac{\alpha_{C_1}}{K_0}\left[1 - \bar{S}_{C_1}(K_0)\right]$$

and $$C'(0) = \left[\frac{d}{ds}C(s)\right]_{s=0}$$

A PARTICULAR CASE:
WHEN ALL REPAIRS FOLLOW EXPONENTIAL TIME DISTRIBUTION:

In this case, we obtain the following Laplace transforms of various flow-state probabilities from equations (48) through (57) by substituting $\bar{S}_i(s) = \frac{\beta_i}{(s+\beta_i)}$, $\forall$ i and s:

$$\bar{P}_0(s) = \frac{1}{E(s)} \qquad \ldots(73)$$

$$\bar{P}_A(s) = \frac{\alpha_A}{E(s)}\left[1 + \frac{\alpha_{C_1}\beta_{C_1}}{(s+\beta_{C_1})(K+\beta_{C_1})}\right]\frac{1}{s+\beta_A} \qquad \ldots(74)$$

$$\bar{P}_B(s) = \frac{\alpha_B}{E(s)}\left[1 + \frac{\alpha_{C_1}\beta_{C_1}}{(s+\beta_{C_1})(K+\beta_{C_1})}\right]\frac{1}{s+\beta_B} \qquad \ldots(75)$$

$$\bar{P}_D(s) = \frac{\alpha_D}{E(s)}\left[1 + \frac{\alpha_{C_1}\beta_{C_1}}{(s+\beta_{C_1})(K+\beta_{C_1})}\right]\frac{1}{s+\beta_D} \qquad \ldots(76)$$

$$\bar{P}_{C_1}(s) = \frac{\alpha_{C_1}}{E(s)(K+\beta_{C_1})} \qquad \ldots(77)$$

$$\bar{P}_{C_1A}(s) = \frac{\alpha_A}{E(s)}\cdot\frac{\alpha_{C_1}}{(s+\beta_{C_1})(K+\beta_{C_1})} \qquad \ldots(78)$$

$$\bar{P}_{C_1B}(s) = \frac{\alpha_B}{E(s)}\cdot\frac{\alpha_{C_1}}{(s+\beta_{C_1})(K+\beta_{C_1})} \qquad \ldots(79)$$

$$\bar{P}_{C_1D}(s) = \frac{\alpha_D}{E(s)}\cdot\frac{\alpha_{C_1}}{(s+\beta_{C_1})(K+\beta_{C_1})} \qquad \ldots(80)$$

$$\bar{P}_C^W(s) = \frac{\alpha_{C_1}\alpha_{C_2}}{(s+w)E(s)(K+\beta_{C_1})} \qquad \ldots(81)$$

and $$\bar{P}_C^R(s) = \frac{w\alpha_{C_1}\alpha_{C_2}}{(s+w)E(s)(K+\beta_{C_1})(s+\beta_C)} \qquad \ldots(82)$$

where, $E(s) = s + \alpha_A + \alpha_B + \alpha_D + \alpha_{C_1} - \left[1 + \frac{\alpha_{C_1}\beta_{C_1}}{(s+\beta_{C_1})(K+\beta_{C_1})}\right]$

$$\times\left[\frac{\alpha_A\beta_A}{s+\beta_A}+\frac{\alpha_B\beta_B}{s+\beta_B}+\frac{\alpha_D\beta_D}{s+\beta_D}\right]-\frac{\alpha_{C_1}\beta_{C_1}}{K+\beta_{C_1}}-\frac{w\alpha_{C_1}\alpha_{C_2}}{(s+w)(K+\beta_{C_1})(s+\beta_C)} \qquad \ldots(83)$$

AVAILABILITY ESTIMATION:

Bared availability of the whole system is given by

$$\overline{P}_{up}(s)=\frac{1}{s+\alpha_A+\alpha_B+\alpha_D+\alpha_{C_1}}\left[1+\frac{\alpha_{C_1}}{s+\alpha_A+\alpha_B+\alpha_D+\alpha_{C_2}}\right]$$

On taking inverse Laplace transform, we get

$$P_{up}(t)=(1+M)\exp.\left\{-\left(\alpha_A+\alpha_B+\alpha_D+\alpha_{C_1}\right)t\right\}-M\exp\left\{-\left(\alpha_A+\alpha_B+\alpha_D+\alpha_{C_2}\right)t\right\} \quad \ldots(84)$$

where, $M=\dfrac{\alpha_{C_1}}{\alpha_{C_2}-\alpha_{C_1}}$

Note that $P_{up}(0)=1$

Also $P_{down}(t)=1-P_{up}(t)$ …(85)

RELIABILITY AND M.T.T.F. EVALUATION:

Reliability of the whole system is given by

$$R(t)=\exp\left\{-\left(\alpha_A+\alpha_B+\alpha_D+\alpha_{C_1}\right)t\right\} \qquad \ldots(86)$$

Also, $M.T.T.F.=\int_0^\infty R(t)\,dt$

$$=\frac{1}{\alpha_A+\alpha_B+\alpha_D+\alpha_{C_1}} \qquad \ldots(87)$$

NUMERICAL ILLUSTRATION:

For a numerical illustration, let us consider the following values of the parameters:

$\alpha_A=0.004$, $\alpha_B=0.008$, $\alpha_{C_1}=0.002$, $\alpha_{C_2}=0.007$, $\alpha_D=0.009$, and t = 0, 1, 2,---.

By using these values in the results (84), (86) and (87), we compute the table- 1, 2 and 3, respectively.

Table-1

t	$\mathbf{P_{up}(t)}$
0	1
1	0.979212
2	0.958843
3	0.938885
4	0.91933
5	0.900169
6	0.881397
7	0.863004
8	0.844984
9	0.82733
10	0.810034

Table-2

t	R(t)
0	1
1	0.977262
2	0.958843
3	0.938885
4	0.91933
5	0.900169
6	0.881397
7	0.863004
8	0.844984
9	0.82733
10	0.810034

Table-3

α_{c_1}	M.T.T.F.
0.01	32.25806
0.02	24.39024
0.03	19.60784
0.04	16.39344
0.05	14.08451
0.06	12.34568
0.07	10.98901
0.08	9.90099
0.09	9.009009
0.10	8.264463
0.11	7.633588

SRESULTS AND DISCUSSION:

In table-1, the author has been computed the value of availability for different values of time t. The corresponding graph has been shown in fig-2. This figure shows that availability of the system decreases slowly with increase in time.

Analysis of table-2 reveals that reliability of the considered system decreases nearly in constant manner with increase in time. Note that there is no sudden jumps in the values of reliability function. Table-3 together yield that MTTF of the system decreases catastrophically in the beginning but thereafter it decreases constantly.

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